Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

9 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A botanist is conducting an experiment which studies the growth of plants.

The heights of the plants are measured on seven different days.

The following table shows the number of days, d, that the experiment has been running and the average height, $h\,\text{cm}$, of the plants on each of those days.

Number of days (d)	2	5	13	24	33	37	42
Average height (h)	10	16	30	59	76	79	82

(a) The regression line of h on d for this data can be written in the form $h = ad + b$.

Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

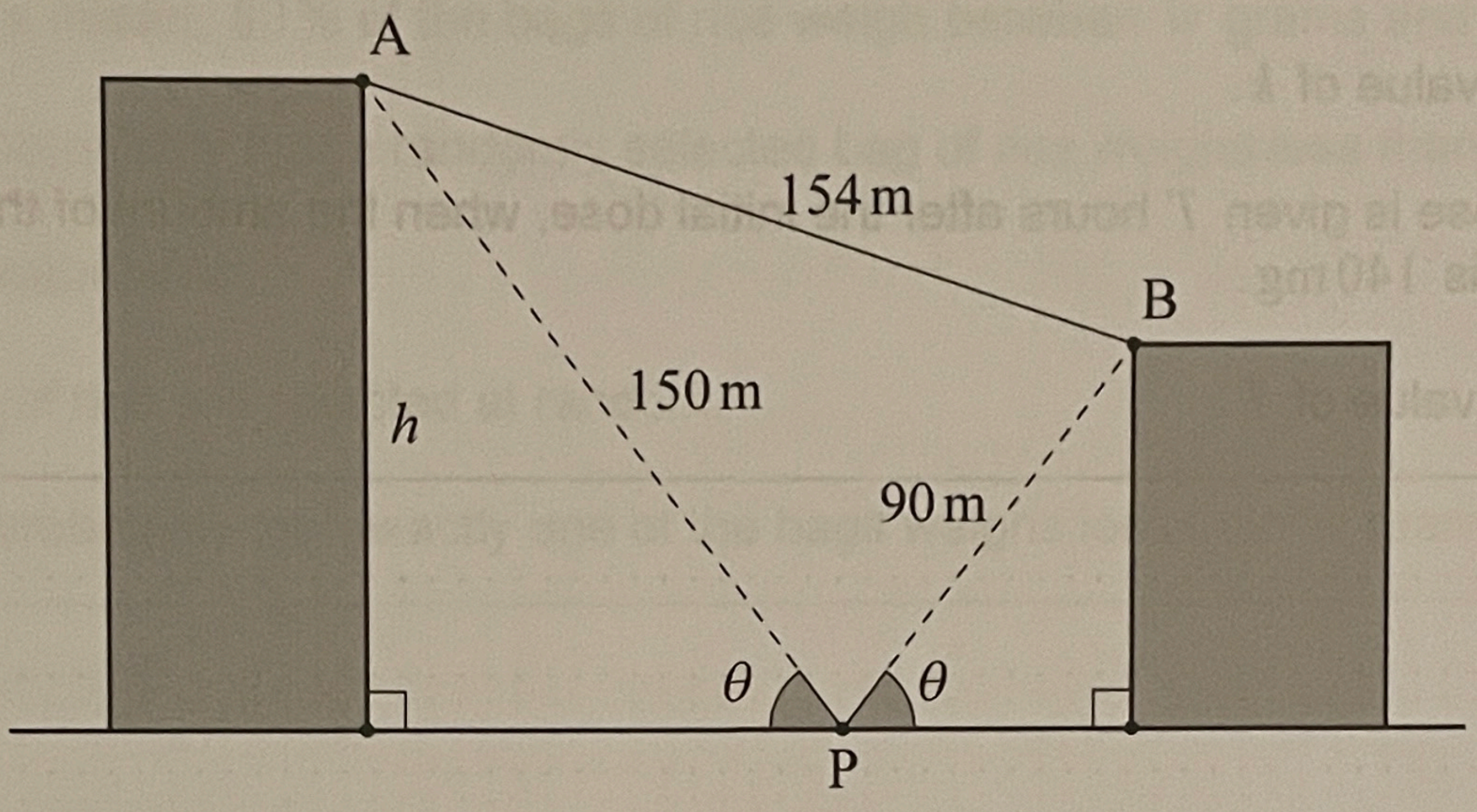
(c) Use your regression line to estimate the average height of the plants when the experiment has been running for 20 days. [2]

2. [Maximum mark: 6]

The following diagram shows two buildings situated on level ground.

From point P on the ground directly between the two buildings, the angle of elevation to the top of each building is θ.

diagram not to scale



The distance from point P to point A at the top of the taller building is 150 metres.

The distance from point P to point B at the top of the shorter building is 90 metres.

The distance between A and B is 154 metres.

(a) Find the measure of $\hat{APB}$. [3]

(b) Find the height, h, of the taller building. [3]

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3. [Maximum mark: 5]

The amount of a drug, in milligrams (mg), in a patient's body can be modelled by the function $A(t) = 500e^{-kt}$, where k is a positive constant and t is the time in hours after the initial dose is given.

(a) Write down the amount of the drug in the patient's body when $t = 0$. [1]

After three hours, the amount of the drug in the patient's body has decreased to $280\,\text{mg}$.

(b) Find the value of k. [2]

The second dose is given T hours after the initial dose, when the amount of the drug in the patient's body is $140\,\text{mg}$.

(c) Find the value of T. [2]

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4. [Maximum mark: 8]

The weights, W grams, of bags of rice packaged in a factory can be modelled by a normal distribution with mean 204 grams and standard deviation 5 grams.

(a) A bag of rice is selected at random.

Find the probability that it weighs more than 210 grams. [2]

According to this model, 80% of the bags of rice weigh between w grams and 210 grams.

(b) Find the probability that a randomly selected bag of rice weighs less than w grams. [2]

(c) Find the value of w. [2]

(d) Ten bags of rice are selected at random.

Find the probability that exactly one of the bags weighs less than w grams. [2]

5. [Maximum mark: 7]

The expansion of $(x+h)^8$, where $h>0$, can be written as $x^8+ax^7+bx^6+cx^5+dx^4+\dots+h^8$, where $a, b, c, d, \dots \in \mathbb{R}$.

(a) Find an expression, in terms of h, for

(i) a;

(ii) b;

(iii) d. [4]

(b) Given that a, b, and d are the first three terms of a geometric sequence, find the value of h. [3]

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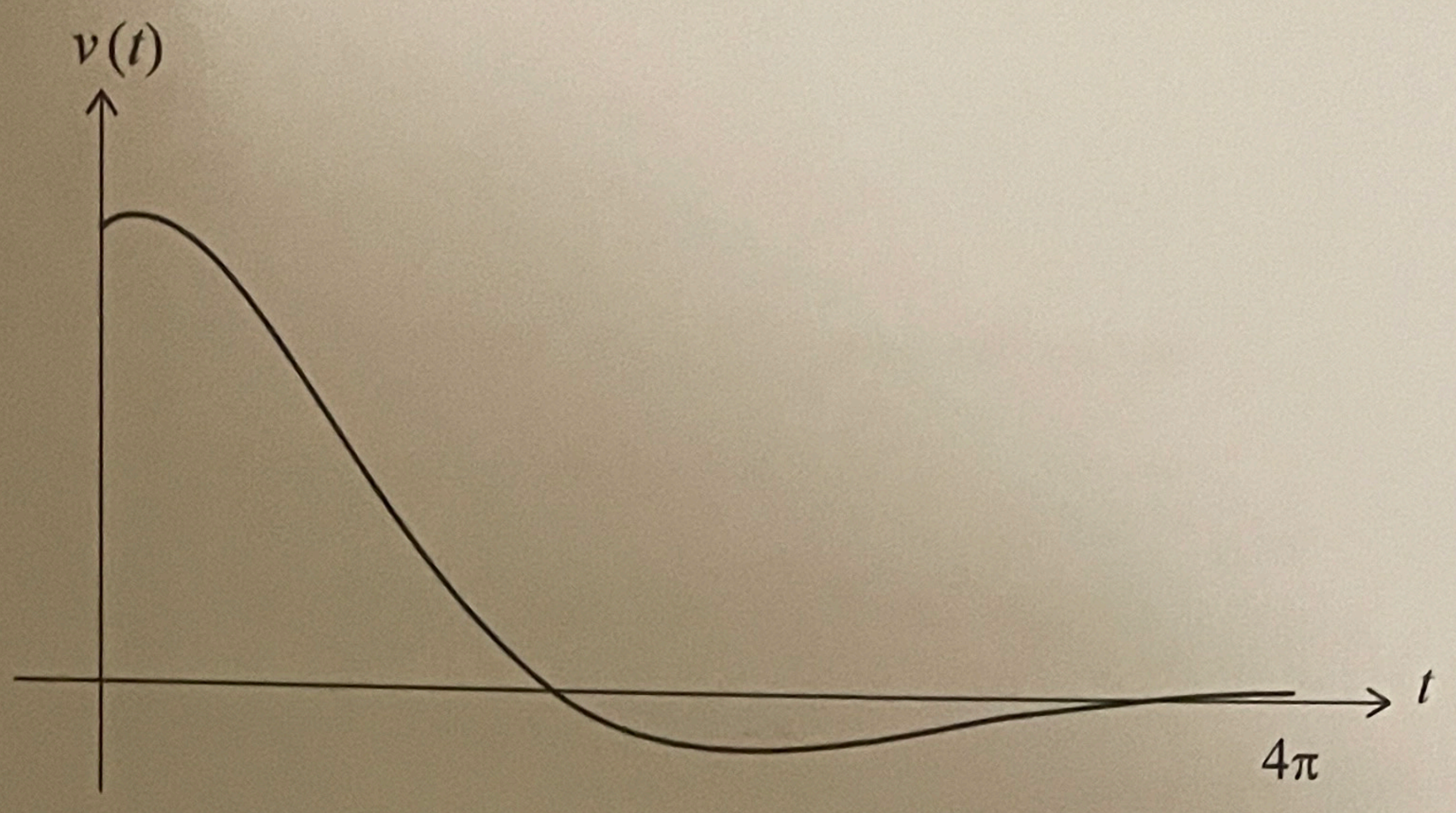
6. [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v\,\text{m}\,\text{s}^{-1}$, at time t seconds is given by $v(t) = 4e^{-\frac{t}{3}}\cos\left(\frac{t}{2} - \frac{\pi}{4}\right)$, for $0 \le t \le 4\pi$. The graph of v is shown in the following diagram.



Let t_1 be the first time when the particle's **acceleration** is zero.

(a) Find the value of t_1. [2]

Let t_2 be the **second** time when the particle is instantaneously at rest.

(b) Find the value of t_2. [2]

(c) Find the distance travelled by the particle between $t = t_1$ and $t = t_2$. [2]

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Please **do not** write on this page.

Answers written on this page will not be marked.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

Consider the function $h(x) = \sqrt{4x-2}$, for $x \geq \frac{1}{2}$.

(a) (i) Find $h^{-1}(x)$, the inverse of $h(x)$, and state its domain.

(ii) Write down the range of $h^{-1}(x)$. [5]

(b) The graph of h intersects the graph of h^{-1} at two points.

Find the x-coordinates of these two points. [3]

(c) Find the area enclosed by the graph of h and the graph of h^{-1}. [2]

(d) Find $h'(x)$. [2]

(e) Find the value of x for which the graph of h and the graph of h^{-1} have the same gradient. [3]

Do **not** write solutions on this page.

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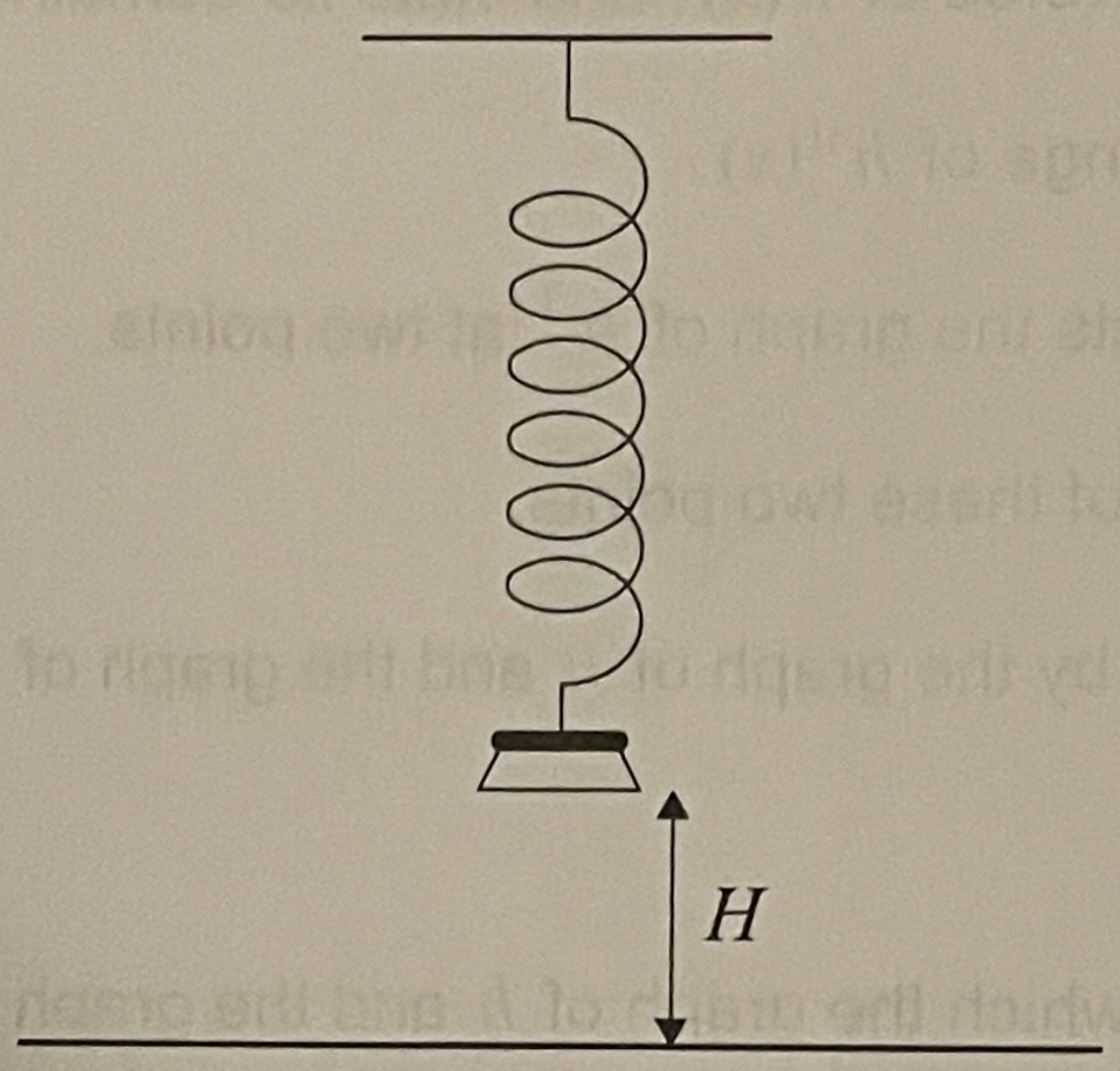
Do **not** write solutions on this page.

8. [Maximum mark: 13]

A weight suspended on a spring is pulled down and released, so that it moves up and down vertically.

The height, H metres, of the base of the weight above the ground can be modelled by the function $H(t) = a\cos(7.8t) + b$, for $a, b \in \mathbb{R}$ and $0 \le t \le 10$, where t is the time in seconds after the weight is released.



(a) Find the period of the function. [2]

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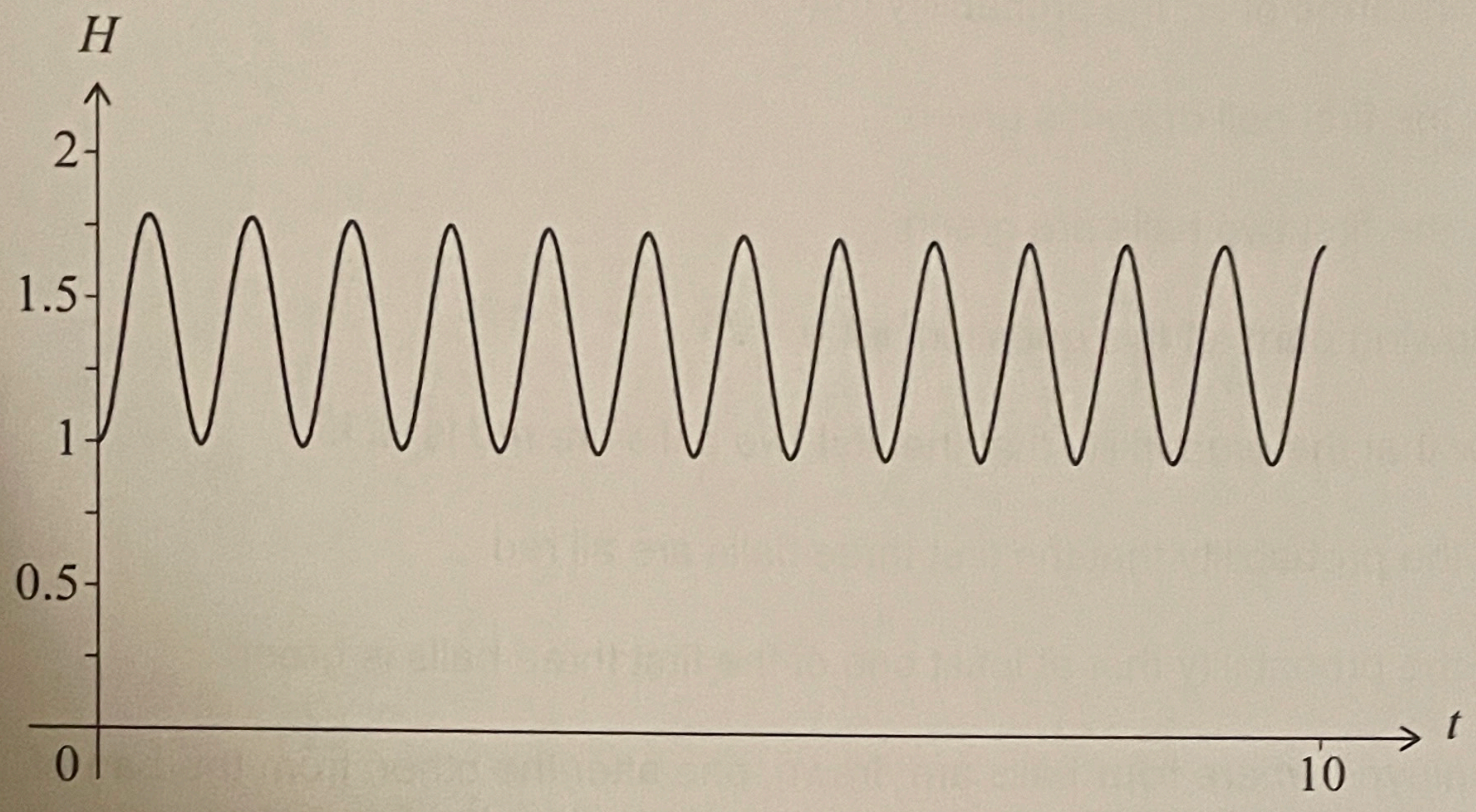
Do **not** write solutions on this page.

(Question 8 continued)

The weight is released when its base is at a minimum height of 1 metre above the ground, and it reaches a maximum height of 1.8 metres above the ground. The graph of H is shown in the following diagram.



(b) Find the value of

(i) a;

(ii) b. [3]

(c) Find the number of times that the weight reaches its maximum height in the first five seconds of its motion. [2]

(d) Find the first time that the base of the weight reaches a height of 1.5 metres. [2]

A camera is set to take a picture of the weight at a random time during the first five seconds of its motion.

(e) Find the probability that the height of the base of the weight is greater than 1.5 metres at the time the picture is taken. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A bag contains n balls. It is known that ten of the balls are green, and the rest of the balls are red. Balls are drawn from the bag, one after the other, without replacement.

(a) Find, in terms of n, the probability that

(i) the first ball drawn is green;

(ii) the first two balls are green. [3]

For the following parts of this question, let $n = 25$.

(b) Show that the probability that the first two balls are red is 0.35. [2]

(c) Find the probability that the first three balls are all red. [2]

(d) Find the probability that at least one of the first three balls is green. [2]

A game is played where **four** balls are drawn, one after the other, from the bag of 25 balls, without replacement. A player earns points based on when the first green ball is drawn. At the end of each game, the four balls are put back in the bag.

A player earns zero points if no green ball is picked, or if the first green ball is picked on the first or second draw.

A player earns 10 points if the first green ball is picked on the third draw and earns 50 points if the first green ball is picked on the fourth draw.

Millie plays this game k times. She finds her score by adding together her points from each game.

(e) Find the least value of k such that Millie's expected score is greater than 100. [6]